Efficiency of Low Power Audio Amplifiers and Loudspeakers

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ABSTRACT

In this paper we look at the load presented to audio amplifiers by real transducers. We consider the power losses in Class-AB and Class-D amplifier topologies, and determine that in order to predict efficiency it is necessary to consider the amplifier/transducer combination. The ability of the class-D amplifier to re-cycle quadrature load current offers new ways to improve efficiency.

INTRODUCTION

Class-D amplifiers are beginning to become viable alternatives to the Class-AB amplifier because of the reducing cost of suitable devices. There is particular interest in developing them for low power applications where their intrinsic efficiency advantages are important. The operation of Class-D amplifiers is very different to Class-AB amplifiers and this has implications for other components in the audio chain.

Conventionally the performance of audio amplifiers is considered using a pure resistive load of four or eight ohms. The origin of this lies in the nominal impedance given to electro-magnetic loudspeakers. The true impedance of an electro-magnetic loudspeaker will vary over the operating frequency range, but the variability between individual real transducers makes the definition of a more realistic 'bench-mark' load impractical. Other audio transducers also have a complex impedance. Transducers which utilise piezo electric elements have a input impedance that is highly reactive. The load presented to the amplifier is almost entirely capacitive.

The power output and efficiency of an amplifier are dependent load impedance and so the resistive load performance may not resemble to the operation of the amplifier in real situations. In this paper we will look at the true load presented to amplifiers by the transducer and derive a more accurate measure of overall efficiency.

GENERAL AMPLIFIER EFFICIENCY

The electrical efficiency of an amplifier is defined as the ratio of the power developed in the load to the power drawn from the DC supply. Using simple linear analysis we can determine the efficiency of amplifier output stages and the dependence of the efficiency on load parameters.

In this section the following symbols are used;

V_o	Output voltage
V_s	Supply rail voltage
R_L	Load resistance
I_{bias}	Class AB quiescent bias current
ϕ	Load phase angle
Z_{load}	Load impedance
L	Class D filter inductance
R_{in}	Resistance of filter inductor
R_{Dson}	'On' state resistance of switching devices
f_s	Class D Switching frequency

Class-AB resistive case

A simple Class-AB output stage is shown in figure 1. A

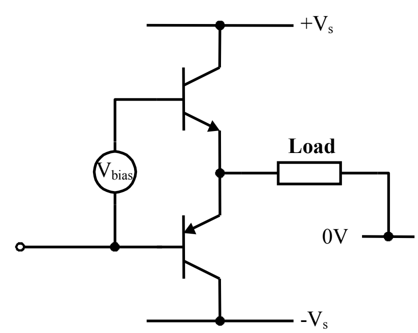

Figure 1 Class-AB output stage

complementary pair of output devices operate over their linear region to amplify the signal. When the devices are operated in the linear region there will be current flowing through them whilst there is a voltage across them, this will give rise to power dissipation, and hence reduce efficiency. The devices also need a quiescent bias to reduce crossover distortion as one device takes over from the other.

We can derive the efficiency of a single ended class AB amplifier driving a resistive load by comparing the power taken from the supply and that developed in the load. Ignoring power loss due to quiescent bias, and assuming a sine wave output, we end up with the familiar equation given by 2.1.

$$\eta_{ab} = \frac{\pi}{4}\frac{\hat{V}_o}{V_s} \tag{2.1}$$

Notice that the efficiency is dependent on the amplitude of the output relative to the supply voltage, rising linearly to a maximum level of 78.5% when the peak output signal is equal to the supply voltage.

The dependence of efficiency on amplitude becomes especially important when considering real music. This is because the average level of a music signal is very much below that of the peak value. Since the average signal will experience an efficiency much less than the peak, the average efficiency when amplifying a music signal will be much lower than that that could be achieved with a simple source e.g. a sine wave.

Class-AB general load

In reality the load driven by the amplifier will not be a pure resistance but be a reactive load. This will have an effect on the efficiency of the amplifier. When the load has a complex impedance there will be a phase difference between the voltage across it and current through it, giving in-phase and quadrature components to the load current. Only the in-phase component produces power in the load. The energy associated with the quadrature component will attempt to flow back and forth between source and load with each cycle.

The design of the Class AB amplifier only allows power flow from source to load, and thus the energy associated with the quadrature component cannot return to the supply. Instead the output devices must dissipate it. This not only reduces the efficiency of the amplifier, it puts extra stress on the devices themselves.

We can develop an expression for the efficiency of a Class-AB amplifier with general load in the same way as before. The efficiency (without bias current) is given in equation 2.2.

$$\eta_{ABc} = \frac{\pi}{4}\frac{\hat{V}_o}{V_s}\cos(\theta) \tag{2.2}$$

We can see how he efficiency of the Class-AB amplifier becomes worse as the load becomes more reactive.

The bias applied to the output stage to prevent crossover distortion results in a quiescent current flowing through both devices. The bias current is independent of output signal level. If we now take into account the bias current, the efficiency of the Class AB output stage becomes that given by equation 2.3.

$$\eta_{ab} = \frac{\hat{V}_o^2 \cos\phi}{4V_s\left(\frac{\hat{V}_o}{\pi} + I_{bias}\left|Z_{load}\right|\right)} \tag{2.3}$$

Class-D amplifiers with resistive loads

Class-D amplifiers encode the audio signal as a pulse width modulated (PWM) signal, which is amplified by a power switching stage. The audio signal is reconstructed, from the PWM, by a low pass filter. A Class-D output stage is shown in figure 2.

If we were to consider the components making up the Class-D amplifiers to be ideal then it would have no sources of loss and therefore 100% efficient (Contrast this with the Class-AB which can only achieve 78% even in the ideal case). Of course in reality there are no ideal components and a practical Class-D amplifier will suffer several sources of loss. The most significant of these are conduction losses arising the resistance associated with the filter inductors and the 'on' state resistance of the output devices (R_{dson} for MOSFETs). The conduction losses are proportional to the resistance and to the square of current. The switching losses of a well designed class-D amplifier are generally insignificant.

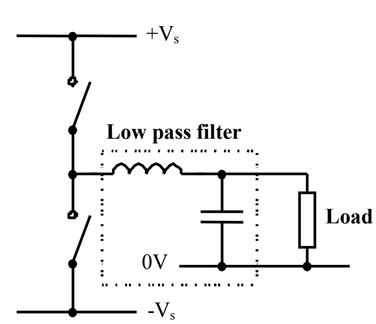

Figure 2 Class-D output stage

The PWM signal contains many different frequency components and an analysis of the Class-D amplifier that attempted to take all of these into account would be unjustifiably complex. For the purposes of this paper we shall simplify it into two components, the audio component and the quiescent switching component. We will assume an output stage based on figure 2, i.e. single ended output and a second order LC low pass filter.

The resistive load efficiency of Class-D amplifier, for a sine wave output, is given in equation 2.5, the derivation of this equation can be found in appendix B.

$$\eta_D = \frac{R_L}{R_L + R_{DSon} + R_{in} + \dfrac{2I_r^2 R_L^2 \left(R_{DSon} + R_{in}\right)}{\hat{V}_o}} \qquad (2.4)$$

where I_r is the RMS ripple current due to the quiescent switching signal and is given by

$$I_r = \frac{V_s}{4\sqrt{3}Lf_s} \qquad (2.5)$$

From equation 2.4 we can identify the contributions of the two signal components. If we ignore the switching components i.e. I_r=0, then the efficiency of the audio component is then dependent not on output amplitude but on the relative size of the load resistance to the total path resistance. The power taken from the supply by the ripple component is independent of output signal level hence the influence it has on the efficiency becomes less significant as the output level increases.

Class-D general load

The action of the Class-D amplifier when presented with a reactive load, and the associated quadrature current, is quite different to that of the Class-AB amplifier.

Because the filter used reconstruct the audio waveform from the switching waveform is reactive, the Class-D amplifier must be capable of dealing with the action of the filter inductor circulating power back and forth. The output stage of the Class-D amplifier is made bi-directional so the energy associated with the quadrature current in the filter is returned to the supply. This ability to cope with reactive energy in an efficient manner also extends to the load. The energy associated with the quadrature load current will be returned to the supply, however the current circulating in the Class-D will still suffer from the conduction losses suffered in the resistive case. The efficiency of the class-D amplifier for a general load is given by equation 2.5.

$$\eta_D = \frac{|Z_L|\cos\phi}{|Z_L|\cos\phi + R_{DSon} + R_{in} + \dfrac{2I_r^2 |Z_L|^2 \left(R_{DSon} + R_{in}\right)}{\hat{V}_o}} \qquad (2.5)$$

Figure 1 shows the efficiency for various load phase angles and output signal levels of Class-AB and Class-D amplifiers. It can clearly be seen that the Class-D has a much better region of high efficiency.

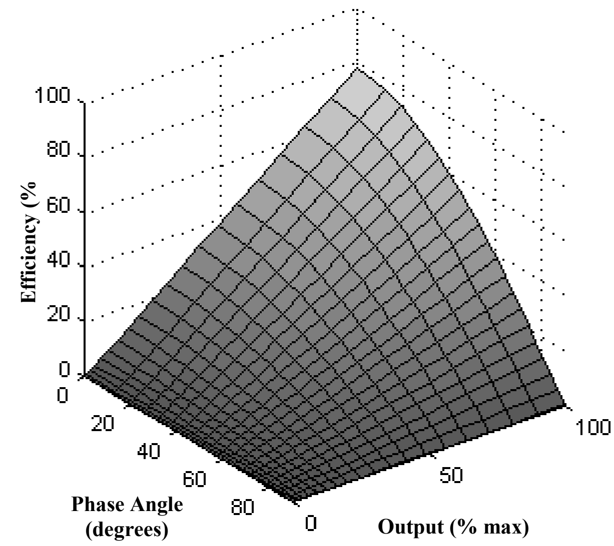

Figure 3a. Class AB Efficiency

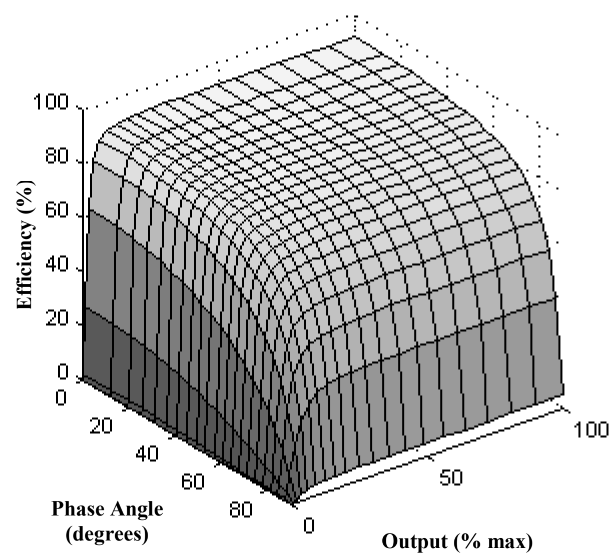

Figure 3b. Class D Efficiency

PIEZO ELECTRIC TRANSDUCERS

Piezo electric audio transducers are used predominantly for ‘tweeters’ in ‘Hi-Fi’ applications or as low cost sounders. However there are piezo transducers available capable of operating down to a few hundred hertz [1], making them suitable for voice and some music applications.

Piezo transducers present a load to the amplifier that is almost entirely capacitive. As we have seen in the previous section highly reactive loads effect the performance of the amplifier.

Two test amplifiers were constructed, one Class-AB and the other Class-D, with output stages following the topology of figure 1 and 2. The amplifiers were designed in a away that allowed the power consumption of the output stages to be measured. The power consumption of the amplifier output stages driving a piezo speaker at 1 KHz, for various output levels was measured. Using the equations so far developed, the power consumption of the amplifiers was also predicted. Both the measured and predicted power consumption is shown in figure 2.

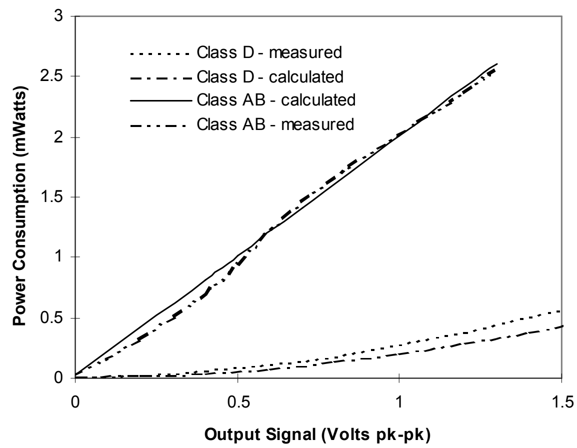

Figure 4. Power Consumption with Piezo Load

Because of the difficulty in practically measuring sound output power, the figure plots power consumption against output signal level. Both amplifiers drove the same piezo speaker, hence will have the same acoustic output for a given output signal level, allowing comparison between the two amplifier topologies.

It can clearly be seen how the class-D amplifier is able to drive the piezo transducer in a much more efficient manner.

The specification of the amplifiers and piezo transducer can be found in appendix B.

ELECTRO-MAGNETIC LOUDSPEAKERS

Conventional methods of calculating the efficiency of electro-magnetic loudspeakers treat the input impedance of the loudspeaker as a pure resistance [2]. Because this leads to an input power that is independent of frequency, the efficiency response follows the same form as the gain response of the speaker [3]. Conventional analysis of loudspeaker efficiency also neglects the effect of the loudspeaker impedance on amplifier efficiency.

We can model the loudspeaker as a lumped element circuit by using electro-mechanical-acoustical analogies [4]. This model is valid over the pistonic range of the cone (limiting the analysis of a typical 8 inch loudspeaker to below 1 KHz [5]). The impedance of a loudspeaker modelled in this way is shown in figure 3.

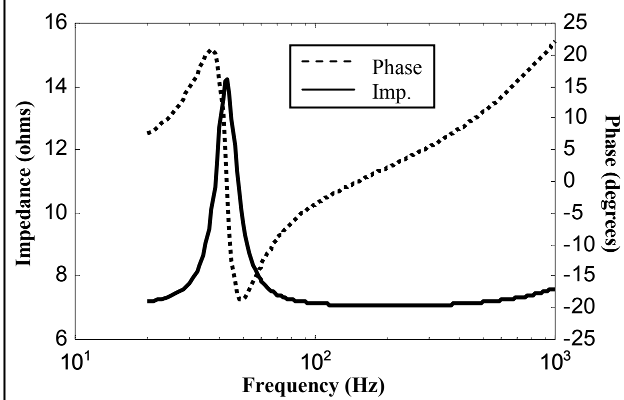

Figure 5. Impedance and phase of electromagnetic loudspeaker

By combining the loudspeaker model with the equations for amplifier efficiency we can predict the true efficiency of loudspeaker/amplifier combinations. Figure 4 shows a comparison of the overall efficiency (conversion of electrical power drawn from supply into sound power) of amplifier/loudspeaker combinations. The true efficiency is shown (when the real impedance of the loudspeaker is presented to the amplifier) and also the efficiency resulting from the conventional simplification, when the input impedance of the loudspeaker is assumed to be resistive.

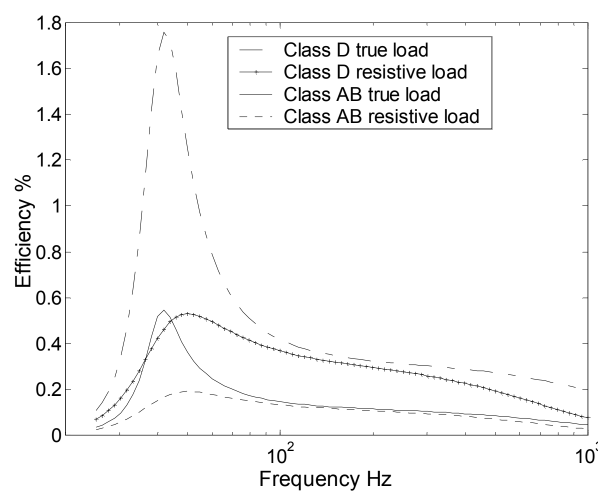

Figure 4. Loudspeaker Efficiency

The parameters of the loudspeaker model and the amplifiers used to produce these plots can be found in appendix B.

Around the first resonance of the loudspeaker (50Hz) the load presented the amplifier is resistive but the input impedance is large and the speaker has a high gain. This is why the models employing the true impedance of the loudspeaker show a much higher efficiency than the resistive load model in this region. It is also evident how the differing models of speaker input impedance alters the frequecy of the peak in efficiency response. Around 200 Hz the load presented to the amplifier is resistive and hence there is little difference between impedance models. At higher frequencies the impedance of the speaker becomes more reactive with the rise in impedance due to the voice coil inductance and the true efficiency becomes different to that predicted by the pure resistnace model.

IMPROVING THE EFFICIENCY OF ELECTRO-MAGNETIC LOUDSPEAKERS

The conventional approach to maximising the efficiency of loudspeakers is to achieve the optimum trade-off between bandwidth and acoustic output over the pass-band [5,6]. Efforts to boost the acoustic output will generally lead to a reduced bandwidth and vice-versa. In the conventional approach efficiency is linked to gain, hence alternations to the loudspeaker that reduce the sensitivity also reduce the efficiency.

By considering the real load presented to the amplifier by the loudspeaker, the gain response and the efficiency response are de-coupled. We can use the model of the loudspeaker and our amplifier output stage models to evaluate the effect of the loudspeaker motor assembly parameters on the system efficiency.

Importantly, because the class-D amplifier is able to re-cycle quadrature energy, a loudspeaker driven by the class-D amplifier can appear reactive without suffering the losses that would result if a class-AB amplifier were used.

The motor assembly

To illustrate the effect on efficiency of loudspeaker variables we will consider the interface between electrical and mechanical sections of the loudspeaker, the motor assembly. There are two main parameters in the simplified model of the motor assembly, the flux density in the air gap and the length of wire within the field. The product of these two terms, known as the 'BL' product relates the current in the voice coil to the mechanical force it produces.

BL product - output

The effect on loudspeaker response of altering the BL product is well known []. Figures 5a and 5b show graphs of sound pressure level (loudspeaker output) against these variables for a constant input signal level, as predicted using the lumped element loudspeaker model. The plots are normalised to the output of the loudspeaker with actual values for flux density and wire length

For the wire length plot the diameter of the wire is assumed constant, as is the diameter of the coil. The overhang of the coil is kept constant, and the extra length of wire is accommodated by assuming the magnetic circuit would be revised to provide the same flux density over the larger area.

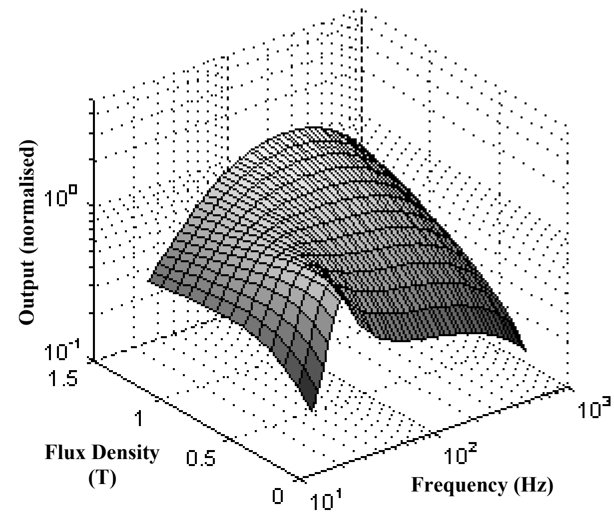

Figure 5a. Output against flux density

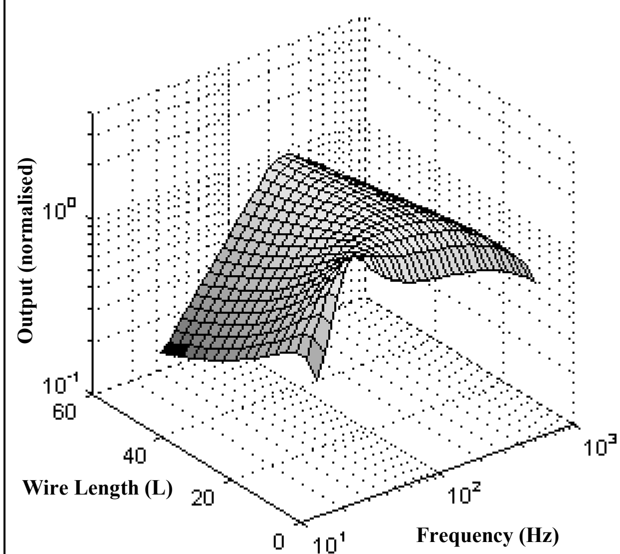

Figure 5b. Output against wire length

Flux density (B) effects only the coupling of the electrical to the mechanical sections. Wire length not only effects the coupling between sections but it also effects electrical resistance and inductance as well as the moving mass. For this reason the shape of the graphs is different.

From figure 4a we can see that a particular value of flux density will produce a maximumly flat response (for a fixed wire length). A higher value of flux density will increase the 'mid-band' response at the expense of the low frequency. Lower values of flux density produce a lower output in the mid-band but a marked peak in output around the first resonance.

From figure 4b we can see how, similar to flux density, there is a particular value of wire length to produce a maximumly flat response (for a fixed flux density). Above this value the mid-band is accentuated, below and a peak in the output around the resonant frequency occurs. However increasing the wire length tends to reduce the sensitivity of the loudspeaker.

How the value of BL is achieved is limited by physical constraints. Since increasing B tends to improve the sensitivity of the speaker whilst increasing L tends to reduce it is best to maximise B and then use the require L to produce the required BL product. The maximum flux density in the air gap is governed by the geometry of the magnet structure, the size of the magnet and the properties of the hard and soft magnetic materials. Values are limited in practice by excessive fringing fields caused by the operation of materials near saturation. Physical size and cost also play a role in magnet assembly design.

BL product – efficiency

Although there is an optimum value of BL product to produce the maximally flat frequency response, this does not reveal the effect of flux density and wire length on the efficiency of the loudspeaker/amplifier combination.

Figure 6 shows how altering the value of flux density effects the overall efficiency of the loudspeaker/amplifier combinations

The improvement in efficiency with increased flux density is clearly shown. Importantly efficiency is improved in areas where the gain response of the loudspeaker (Figure 4a) is reduced. This is in contrast to the traditional approach, which leads to a reduction in efficiency if the gain response is reduced.

The improved intrinsic efficiency of the Class-D amplifier compared to the Class-AB amplifier accounts for the much higher peak efficiency of the Class-D/loudspeaker combination, whilst the difference in shape of the plots is due to the Class-D amplifier being able to recycle quadrature power.
Figure 9 shows the effect of wire length on efficiency for the amplifier/loudspeaker combinations

Unlike flux density, alterations to L effect many other parameters of the speaker. Electrical inductance, resistance and moving mass are also dependent on the length of wire. If we think in terms of our redefined efficiency we note that although inductance and mass will alter the response of the speaker they are reactive components and will therefore not create losses for the

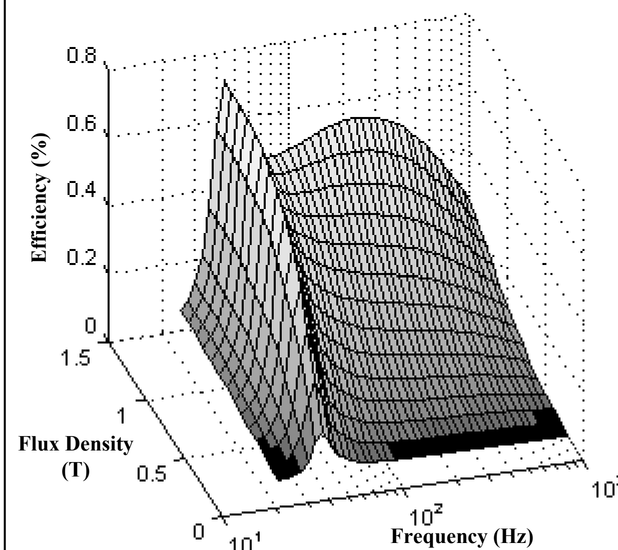

Figure 6a. Class AB Efficiency against flux density

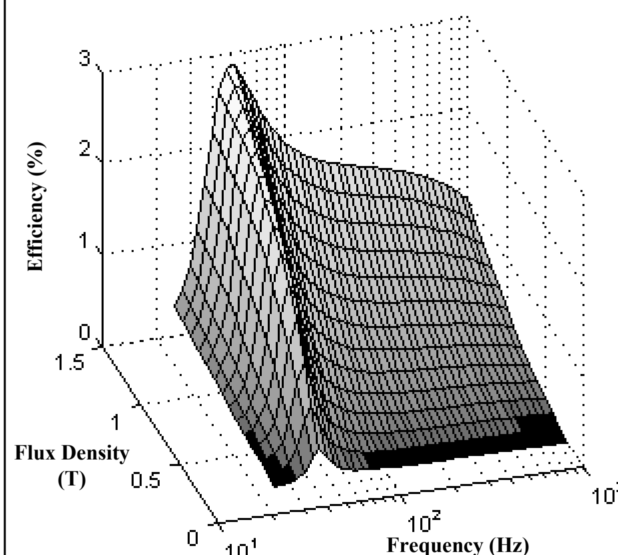

Figure 6b. Class D Efficiency against flux density

Class-D amplifier/loudspeaker combination. The increased resistance will be a source of loss for both combinations.

Figure 8 shows the efficiency, integrated over frequency, of the loudspeaker/amplifier combinations and provides a direct comparison of their performance.

Whilst the Class-D amplifier/loudspeaker combination displays a clear improvement in efficiency as the coil length is increased, the same cannot be said for the Class-AB combination where there is only slight improvement.

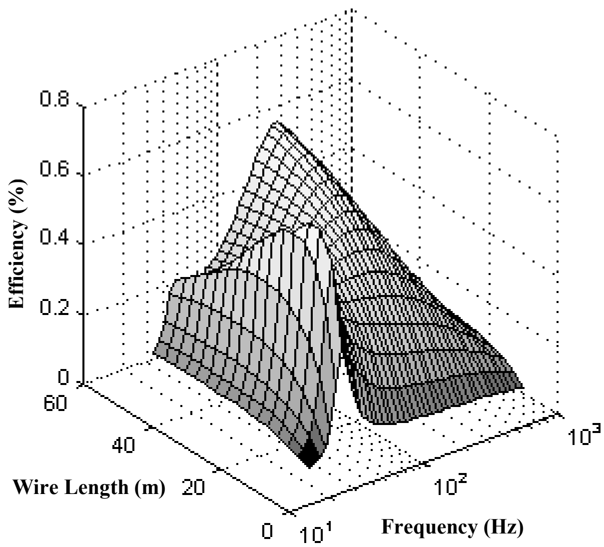

Figure 7a. Class AB Efficiency against wire length

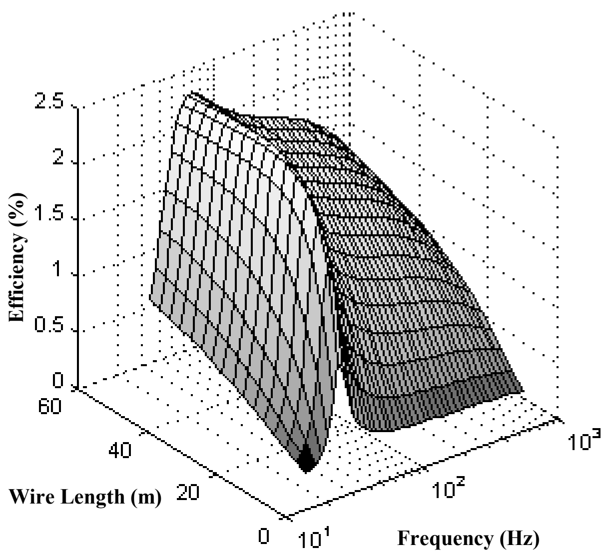

Figure 7b. Class D Efficiency against wire length

Limitations of analysis

The analysis of the loudspeaker only valid over the low frequency region (<1KHz). Even over this region the gain response of the loudspeaker is effected. Increasing the mass of the loudspeaker would reduce the sensitivity at high frequencies, hence any improvements in efficiency obtained over the low frequency range are unlikely to extend to higher frequencies.

The pre-power stage circuitry associated with each amplifier type has not been considered. This can be

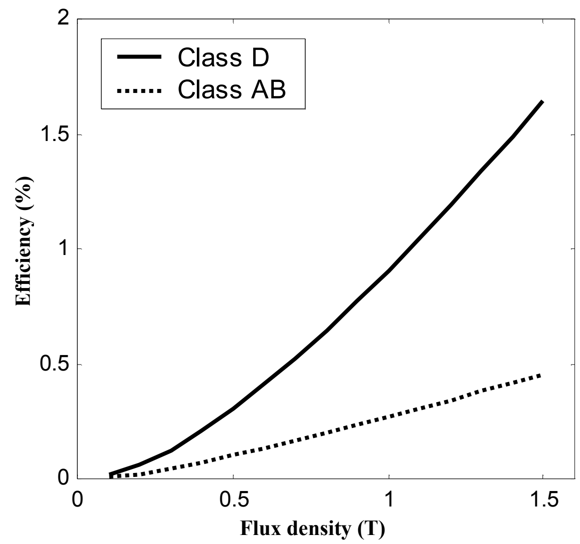

Figure 6a. Efficiency averaged-over-frequency against flux density

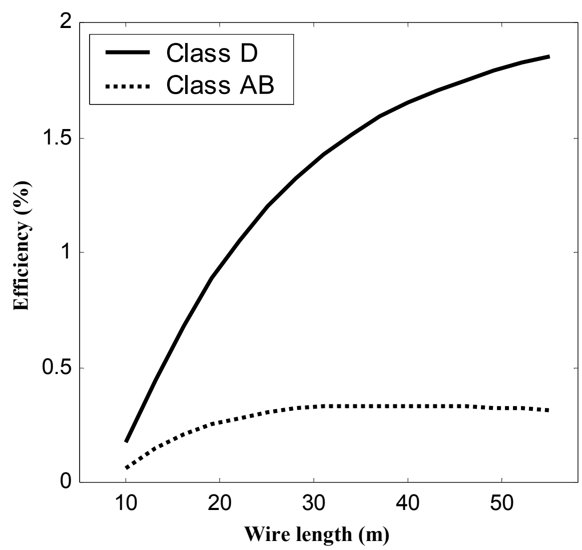

Figure 6b. Efficiency averaged-over-frequency against wire length

significant particularly in the case of the Class-D amplifier, where generation of the PWM and switch driver circuits are required.

CONCLUSION

The load presented to an amplifier is an important factor on determining efficiency. The ability of the class-D amplifier to recover the energy associated with quadrature load current can lead to an improved efficiency when the load is reactive.

The efficiency of the sound reproduction process is dependent on the amplifier as well as the transducer. The combination of loudspeaker and amplifier must be considered to predict the efficiency.

Modifications to improve the efficiency of the loudspeaker often make the load presented to the amplifier more reactive. In these situations a greater benefit is seen with class-D amplifiers. There is potential to design high efficiency loudspeakers based on the characteristics of the Class-D amplifier

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APPENDIX A

Derivation of Class-D efficiency

We have simplified the PWM signal into two components, an audio frequency component and the quiescent switching signal. This is valid since in a practical amplifier the power loss due to the switching harmonics are small compared to those associated with the audio signal. Hence although the switching harmonics deviate from the quiescent state when a modulating signal is present, this change is insignificant compared to the power loss due to the modulating signal itself.

We will assume that the upper and lower devices have matched on state resistance. This enables us to make the simplification that, although the action of the switching circuit is to alternate the current between upper and lower devices at the switching frequency, the current will flow through a resistance equivalent to the 'on' state resistance of each device all the time.

The audio frequency component flows through the device 'on' state resistance, the filter inductor resistance and through the load impedance. Assuming a simple sine wave modulating signal the power taken from the load due to the audio frequency is given by:

$$P_{au} = \frac{\hat{V}_o^2}{2R_L^2}\left(R_{DSon} + R_{in} + R_L\right)$$

The switching signal is a square wave of magnitude equal to the supply rails. This waveform is applied across the filter inductor hence the corresponding current waveform is triangular. The RMS value of the quiescent current waveform is given by:

$$I_{rip} = \frac{V_+}{\sqrt{3}.4.f_s L}$$

The quiescent current flows through the device resistance, the inductor resistance and to ground through the filter capacitor. Assuming the capacitor to be ideal the power drawn from the supply by the quiescent component is then:

$$P_{rip} = I_{rip}^2\left(R_{DSon} + R_{in}\right)$$

The power developed in the load is solely due to the audio component and given by:

$$P_L = \frac{\hat{V}_o^2}{2R_L}$$

Combining these equations are re-arranging we can produce a equation for efficiency:

$$\mu_D = \frac{R_L}{R_{DSon} + R_{in} + R_L + \frac{2I_{rip}^2 R_L^2}{\hat{V}_o^2}\left(R_{DSon} + R_{in}\right)}$$

For a reactive load the equation becomes:

$$\eta_D = \frac{\left|Z_L\right|\cos\phi}{\left|Z_L\right|\cos\phi + R_{DSon} + R_{in} + \frac{2I_r^2\left|Z_L\right|^2\left(R_{DSon} + R_{in}\right)}{\hat{V}_o^2}}$$

APPENDIX B

Piezo Load Test Parameters

The parameters of the test output stages used to measure power consumption against output level with a piezo load are as follows. These parameters were used with the models to predict power consumption.

Class-AB

V_s	5V
I_{bias}	1.3 mA

Class-D

V_s	5V
R_{Dson}	
R_{in}	
L	
f_s	

Piezo load assumed to be a 1uf capacitance in series with a 10 ohm resistance

Electro-magnetic load test Parameters

The parameters of the loudspeaker and amplifiers used to produce the gain and efficiency plots are as follows.

Class-AB

V_s
V_o
I_{bias}

Class-D

V_s
V_o
R_{Dson}
R_{in}
L
f_s

Loudspeaker

Coil resistance
Coil inductance
Flux density

Coil length in field
Suspension loss
Compliance
Moving mass
Cone effective diameter
Voice coil diameter
Magnetic field depth
Coil depth

Radiation impedance assumed to be that of a circular disc in infinite baffle, using approximation given in [9].